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LAMINAR HEAT TRANSFER TO A STEADY COUETTE FLOW BETWEEN PARALLEL PLATES

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Abstract—An exact analytical solution of the energy equation is given for the laminar, steady state Couette flow of an incompressible viscous fluid between parallel plates. Thermophysical properties of the fluid are assumed to be temperature independent. Four cases of boundary conditions are considered. These include unit temperature steps at one or both plates or a unit temperature step at one plate, the other plate being completely insulated. The spacial development of the temperature profiles is set forth graphically. Variation of the local Nusselt number with longitudinal distance and limiting values for the region of full thermal stabilization are reported.

NOMENCLATURE

a, thermal diffusivity of the fluid;
A_n, constant defined in equation (27);
B_n, constant defined in equation (22);
c_p, specific heat at constant pressure;
C_n, constant defined in equation (29);
D_n, constant defined in equation (30);
E_n, constant defined in equation (35);
f, function of the η variable, equation (21);
F_n, constant defined in equation (36);
g, function of the ξ variable, equation (21);
G_n, constant defined in equation (39);
h, heat-transfer coefficient;
H, distance between the plates;
J_ν, Bessel function of the first kind, νth order;
k, thermal conductivity;
$Nu(\xi)$, local Nusselt number;
Nu_{sp}, local Nusselt number at the stationary plate;
Nu_{mp}, local Nusselt number at the moving plate;
$Nu(\infty)$, limiting value of the Nusselt number;
Pe, Péclet number;
T, temperature;
T_0, temperature of the fluid at $z = 0$;
T_1, temperature at the wall;
T_b, "cup-mixing" or bulk temperature;
u_z, point velocity in the z-direction;
U, velocity of the moving plate;
y, distance perpendicular to the plates;
z, distance along the wall.

Greek symbols

α_n, eigenvalues of the b.v. problems considered;
Γ, gamma function;
ξ, dimensionless longitudinal distance;
η, dimensionless transverse distance;
ϑ, dimensionless temperature;
ϑ_w, dimensionless temperature at the wall;
ϑ_b, dimensionless bulk temperature;
ρ, density of the fluid.

INTRODUCTION

HEAT or mass transport which frequently occurs in fluid flow systems involving moving boundaries plays a significant role in many industrial applications. The complexity of the mathematical solution of the problem depends up to a great extent upon the character of the fluid velocity field in the particular geometry under consideration.

Beek and Bakker [1, 2] calculated mass-transfer coefficients for Newtonian fluid flow with a plane moving interface along which a boundary layer develops. Their paper [1] also includes a solution for the semi-infinite Couette flow bounded by a moving plate across which mass transfer takes place. Couette flow in confined geometries, e.g. between parallel plates, Fig. 1 has been adopted as a model of the flow and heat-transfer behaviour of lubricants in journal bearings. Vogelpohl [3] calculated the temperature distribution due to irreversible mechanical energy dissipation in bearings assuming a constant temperature at both walls of the gap. In a subsequent paper [4] the same problem has been solved for the case where both walls were completely insulated. Hudson [5] and later on Hudson and Bankoff [6] calculated the temperature distribution in a narrow passage formed by an endless belt in uniform translatory motion parallel to a stationary wall. The fluid was assumed to enter the

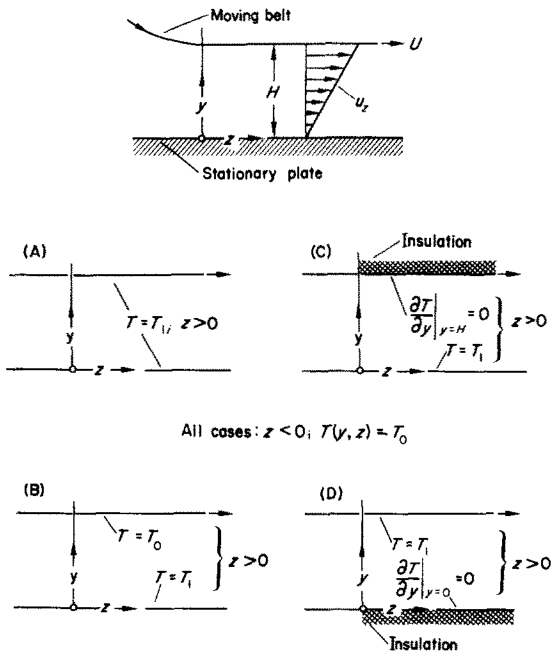

FIG. 1. Location of the coordinate system and boundary conditions.

channel with a uniform temperature, both walls were held at an equal temperature, different from that of the entering fluid. In addition, imposed to the simple Couette flow, a constant pressure gradient along the duct was assumed to exist. In their paper, the first five eigenvalues and eigenfunctions of the corresponding Sturm–Liouville problem were found by numerical integration. If there is no pressure gradient acting along the duct however, the convective transport of the fluid is due entirely to the motion of the belt and the solution of the corresponding boundary value problem is obtained in a closed form.

It is the purpose of this paper to derive expressions for the temperature distributions and Nusselt numbers corresponding to physical models characterized by boundary conditions which are different from that treated in the previous investigations. These may also be more realistic in some other industrial applications of the Couette flow model accompanied with heat or mass transfer.

THE STATEMENT OF THE PROBLEM

A parallel plate channel is formed by a stationary lower wall and a uniformly moving upper plate which may either be an endless belt or a fictitious plane normal to the blade of a rotational, thin-film heat exchanger, see Fig. 1. Assuming temperature independent thermophysical properties of the transported fluid, the upper plate induces a uniform velocity gradient in the channel. This requires the shear stress to remain at a constant value across the duct which in turn means that, the corresponding linear velocity distribution will hold for the case of simple non-Newtonian models of rheological behaviour as well as for the Newtonian linear relation between the stress and rate-of-deformation tensors. Therefore, the velocity distribution in the channel is

$$u_z = \frac{U}{H} \cdot y. \tag{1}$$

Neglecting the term representing longitudinal molecular transport of heat, viscous dissipation as well as other heat generation or depletion terms, the steady-state form of the energy equation in Cartesian coordinates becomes, [7]:

$$\rho c_p u_z \frac{\partial T}{\partial z} = k \frac{\partial^2 T}{\partial y^2}. \quad (2)$$

Substituting from equation (1) for the velocity component u_z into the energy equation (2) along with the definition of the thermal diffusivity $a = k/(\rho \,.\, c_p)$, there is

$$y \,.\, \frac{U}{H}\frac{\partial T}{\partial z} = a \frac{\partial^2 T}{\partial y^2}. \quad (3)$$

Four cases of boundary conditions had been considered; the arrangement of the corresponding physical models is shown schematically in Figs. 1(A–C). At the entrance of the channel, the fluid has a uniform temperature:

$$T = T_0, \qquad 0 \leqslant y \leqslant H, \qquad z \leqslant 0. \quad (4)$$

This boundary condition is common for all the four cases considered in this paper, The remaining boundary conditions may be summarized as follows:

Case A (*Fig.* 1*a*). Both plates of the channel maintained at a constant temperature T_1.

$$\begin{matrix} T = T_1; & y = 0 \\ T = T_1; & y = \mathrm{H} \end{matrix} \quad z > 0 \cdot \quad (5)$$

Equation (3) along with the boundary condition in equations (4–5) is obtained as a particular case of the boundary value problem in [6] for a zero pressure gradient.

Case B (*Fig.* 1*b*). Stationary plate maintained at a constant temperature T_1 different from that of the entering fluid.

$$\begin{matrix} T = T_1: & y = 0 \\ T = T_0; & y = \mathrm{H} \end{matrix} \quad z > 0 \quad (6)$$

Case C (*Fig.* 1*c*). Zero heat flux at the upper moving plate· This may be a consequence of a thermal insulation or negligible value of the coefficient of heat transfer into the adjacent medium.

$$\begin{matrix} T = T_1; & y = 0 \\ \partial T/\partial y = 0; & y = \mathrm{H} \end{matrix} \quad z > 0. \quad (7)$$

Case D (*Fig.* 1*d*). Zero heat flux at the stationary plate.

$$\begin{matrix} \partial T/\partial y = 0; & y = 0 \\ T = T_1; & y = \mathrm{H} \end{matrix} \quad z > 0. \quad (8)$$

The local heat-transfer coefficient expressed in terms of the dimensionless Nusselt number is

$$Nu = hH/k = \frac{-H\partial T/\partial y|_w}{T_w - T_b} \quad (9)$$

where the bulk or cup-mixing temperature T_b is defined as

$$T_b = \int_0^H u_z T \mathrm{d}y \Bigg/ \int_0^H u_z \,\mathrm{d}y = \frac{2}{\mathrm{H}^2}\int_0^H Ty\,\mathrm{d}y. \quad (10)$$

ANALYSIS

Introducing the following dimensionless variables and the Péclet group

$$\vartheta = \frac{T_1 - T}{T_1 - T_0}; \quad \eta = y/H$$
$$\xi = \frac{z}{H \,.\, Pe}; \qquad Pe = \frac{UH}{a}. \quad (11)$$

Equations (3–10) reduce to

$$\eta \frac{\partial \vartheta}{\partial \xi} = \frac{\partial^2 \vartheta}{\partial \eta^2} \quad (12)$$

$$\vartheta = 1; \qquad 0 \leqslant \eta \leqslant 1; \qquad \xi \leqslant 0. \quad (13)$$

Case A

$$\left.\begin{matrix} \vartheta = 0; & \eta = 0 \\ \vartheta = 0; & \eta = 1 \end{matrix} \quad \xi > 0. \right\} \quad (14)$$

Case B

$$\left.\begin{matrix} \vartheta = 0; & \eta = 0 \\ \vartheta = 1; & \eta = 1 \end{matrix} \quad \xi > 0. \right\} \quad (15)$$

Case C

$$\left.\begin{matrix} \vartheta = 0; & \eta = 0 \\ \partial\vartheta/\partial\eta = 0; & \eta = 1 \end{matrix} \quad \xi > 0. \right\} \quad (16)$$

Case D

$$\left.\begin{aligned} \partial\vartheta/\partial\eta = 0; \quad \eta = 0 \\ \vartheta = 0; \quad \eta = 1 \end{aligned} \quad \xi > 0. \right\} \tag{17}$$

$$Nu = \frac{-\partial\vartheta/\partial\eta|_w}{\vartheta_w - \vartheta_b} \tag{18}$$

$$\vartheta_b = \frac{T_1 - T_b}{T_1 - T_0} = 2\int_0^1 \vartheta\eta\,\mathrm{d}\eta. \tag{19}$$

Separating the variables on assuming the solution of the form

$$\vartheta = f(\eta)g(\xi) \tag{20}$$

Equation (12) becomes

$$\frac{1}{g}\frac{\mathrm{d}g}{\mathrm{d}\xi} = -\frac{9}{4}\alpha^2 = \frac{1}{\eta f}\frac{\mathrm{d}^2 f}{\mathrm{d}\eta^2} \tag{21}$$

where $-9\alpha^2/4$ is the separation constant. The solution of the resulting ordinary differential equations is straightforward and one obtains for the general solution:

$$\vartheta(\eta, \xi) = \sqrt{(\eta)}\sum^{\infty}[A_n J_{\frac{1}{3}}(\alpha_n\eta^{\frac{3}{2}}) + B_n J_{-\frac{1}{3}}(\alpha_n\eta^{\frac{3}{2}})]\exp(-\tfrac{9}{4}\alpha_n^2\xi). \tag{22}$$

Case A

From equation (14) $\vartheta(0, \xi) = 0$, wherefrom $B_n = 0$ and equation (22) reduces to

$$\vartheta(\eta, \xi) = \sqrt{(\eta)}\sum_{n=0}^{\infty} A_n J_{\frac{1}{3}}(\alpha_n\eta^{\frac{3}{2}})\exp(-\tfrac{9}{4}\alpha_n^2\xi). \tag{23}$$

The boundary condition at the moving plate $\vartheta(1, \xi) = 0$ yields

$$\sum_{n=0}^{\infty} A_n J_{\frac{1}{3}}(\alpha_n)\exp(-\tfrac{9}{4}\alpha_n^2\xi) = 0. \tag{24}$$

This requires the eigenvalues α_n to be positive roots of $J_{\frac{1}{3}}(\alpha) = 0$. Since it is not difficult to prove using equations (14) and (21) that the eigenfunction corresponding to the first eigenvalue $\alpha_0 = 0$ vanishes, the general solution of the Case A is

$$\vartheta(\eta, \xi) = \sqrt{(\eta)}\sum_{n=1}^{\infty} A_n J_{\frac{1}{3}}(\alpha_n\eta^{\frac{3}{2}})\exp(-\tfrac{9}{4}\alpha_n^2\xi). \tag{25}$$

The constants A_n are found from the boundary conditions at the duct entrance, equation (13):

$$T = \sum_{n=1}^{\infty} A_n\eta^{\frac{3}{2}} J_{\frac{1}{3}}(\alpha_n\eta^{\frac{3}{2}}). \tag{26}$$

Multiplying equation (26) by $\eta^{\frac{3}{2}}J_{\frac{1}{3}}(\alpha_m)\,\mathrm{d}\eta$, integrating from 0 to 1 and making use of the orthogonal property of the eigenfunctions gives for the A_n:

$$A_n = \frac{\int_0^1 \eta^{\frac{3}{2}}J_{\frac{1}{3}}(\alpha_n\eta^{\frac{3}{2}})\,\mathrm{d}\eta}{\int_0^1 \eta^2 J_{\frac{1}{3}}^2(\alpha_n\eta^{\frac{3}{2}})\,\mathrm{d}\eta} = \frac{[(\alpha_n/2)^{\frac{1}{3}}\Gamma(2/3)J_{-\frac{1}{3}}(\alpha_n) - 1]}{(\alpha_n/2)J_{-\frac{2}{3}}(\alpha_n)} \tag{27}$$

and makes thus the expression for the temperature distribution complete. Calculation of the temperature profiles has been programmed and performed on the IBM 7094 digital computer of the University of Toronto. Eigenvalues for this and all the following boundary value problems were taken from [8]. Figure 2 shows some temperature profiles for the symmetrically heated channel. It has been mentioned earlier, that this situation is equivalent to the boundary value problem treated in [6] for the special case of a negligible pressure gradient. Unfortunately, a rigorous comparison of the numerical results was not possible since the constants A_n are missing in the material tabulated in [6]. A qualitative check against some temperature profiles presented in [6] in graphical form brought out satisfactory agreement.

In order to calculate the local Nusselt groups, the bulk temperature is obtained on inserting the expression for $\vartheta(\eta, \xi)$ from equation (25) into (19) and integrating from 0 to 1. The dimensionless heat flux, having different values at both plates, yields different values of the Nusselt number· Performing all the calculations outlined earlier gives for the Nusselt number Nu_{mp} at the moving plate:

$$Nu_{mp} = -\tfrac{9}{2}\Gamma(1/3)\frac{\sum_{n=1}^{\infty} C_n\exp(-\frac{9}{4}\alpha_n^2\xi)}{\sum_{n=1}^{\infty} D_n\exp(-\frac{9}{4}\alpha_n^2\xi)} \tag{28}$$

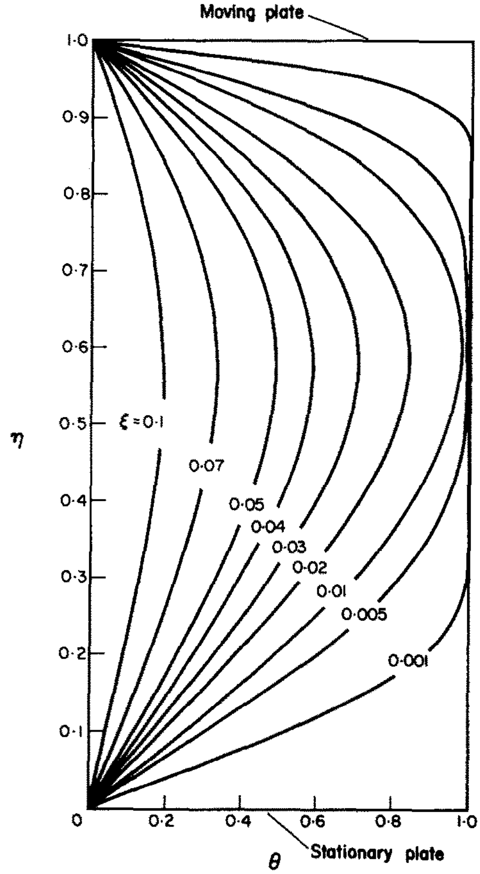

FIG. 2. Dimensionless temperature profiles, boundary condition, Case A.

where

$$C_n = 1 - (\alpha_n/2)^{2/3}\Gamma(2/3)J_{-1/3}(\alpha_n) \qquad (29)$$

$$D_n = \frac{C_n[1 - \Gamma(1/3)(\alpha_n/2)^{2/3}J_{-2/3}(\alpha_n)]}{(\alpha_n/2)^{4/3}J_{-2/3}(\alpha_n)}. \qquad (30)$$

The Nusselt group Nu_{sp}, characterizing the heat transfer at the stationary plate is obtained in a similar way:

$$Nu_{sp} = \frac{9}{2}\,\frac{\sum_{n=1}^{\infty} \frac{C_n}{(\alpha_n/2)^{4/3}J_{-2/3}(\alpha_n)} \exp(-\frac{9}{4}\alpha_n^2\xi)}{\sum_{n=1}^{\infty} D_n \exp(-\frac{9}{4}\alpha_n^2\xi)} \qquad (31)$$

C_n and D_n having the same meaning as in equation (28). The variation of Nu_{mp} and Nu_{sp} with the dimensionless longitudinal distance ξ is shown in Fig. 3. Since the heat fluxes at both plates are opposing each other, the Nusselt group definition in equation (18) results in negative values of Nu_{sp}. In order to calculate the total Nusselt number $|Nu_{sp}| + |Nu_{mp}|$ which is a measure of the total heat quantity entering the fluid across the solid interfaces, both Nusselt groups are plotted positive in Fig. 3.

Case B

As far as the calculation of the temperature profiles for these and all the remaining boundary conditions is essentially the same as in case A, no details of the procedure will be given in the following text. The expression for the temperature distribution has the form:

$$\vartheta(\eta, \xi) = \eta + \frac{\sqrt{(\eta)}}{\Gamma(1/3)} \sum_{n=1}^{\infty} \frac{J_{1/3}(\alpha_n \eta^{3/2})}{(\alpha_n/2)^{4/3}J^2_{-2/3}(\alpha_n)} \times \exp(-\tfrac{9}{4}\alpha_n^2\xi). \qquad (32)$$

The eigenvalues α_n are again positive roots of $J_{1/3}(\alpha) = 0$. Figure 4 shows temperature profiles for some values of the dimensionless longitudinal distance ξ, Fig. 5 presents the variation of the Nusselt numbers with ξ. The Nusselt number at the moving plate is

$$Nu_{mp} = \frac{3 + 9\sum_{n=1}^{\infty} E_n \exp(-\frac{9}{4}\alpha_n^2\xi)}{1 - 2\sum_{n=1}^{\infty} F_n \exp(-\frac{9}{4}\alpha_n^2\xi)} \qquad (33)$$

whereas at the stationary plate,

$$Nu_{sp} = \frac{3 + 9\sum_{n=1}^{\infty} E_n \exp(-\frac{9}{4}\alpha_n^2\xi)}{2 + 2\sum_{n=1}^{\infty} F_n \exp(-\frac{9}{4}\alpha_n^2\xi)} \qquad (34)$$

The constants E_n and F_n in equations (33–34) are

$$E_n = 1/[\Gamma(1/3)(\alpha_n/2)^{2/3}J_{-2/3}(\alpha_n)] \qquad (35)$$

and

$$F_n = \frac{1 - (\alpha_n/2)^{2/3}\Gamma(1/3)J_{-2/3}(\alpha_n)}{\Gamma^2(1/3)(\alpha_n/2)^{10/3}J^2_{-2/3}(\alpha_n)}. \qquad (36)$$

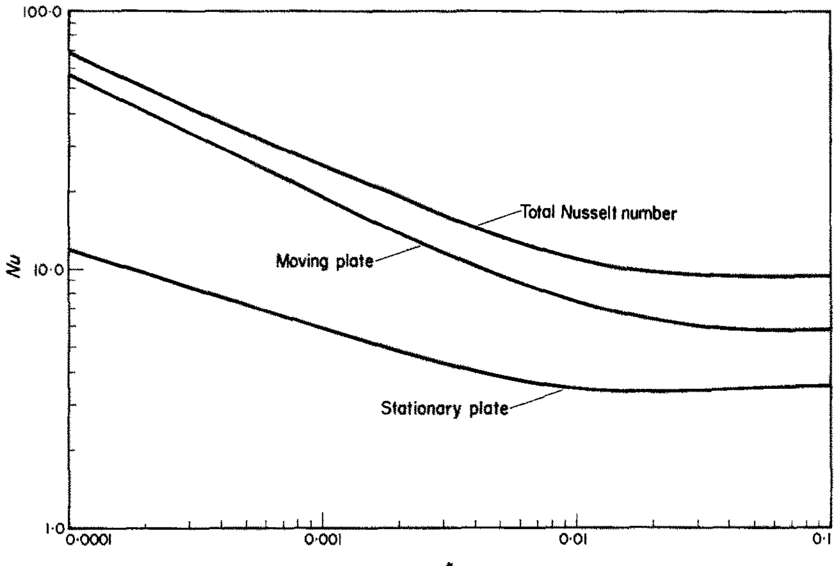

FIG. 3. Variation of the local Nusselt number with longitudinal distance, boundary condition Case A.

In Case A, zero heat fluxes at both plates are approached with increasing values of ξ. Thermal equilibrium in Case B is characterized by dimensionless heat fluxes having the same direction and unit magnitude. Consequently, a steady heat flow and linear temperature distribution across the channel is approached far from the entrance to the channel.

Case C

The expression for the temperature profile is:

$$\vartheta(\eta, \xi) = \frac{\sqrt{(\eta)}}{\Gamma(1/3)} \sum_{n=1}^{\infty} (2/\alpha_n)^{\frac{2}{3}} \times \frac{J_{\frac{1}{3}}(\alpha_n \eta^{\frac{3}{2}})}{J_{\frac{1}{3}}^2(\alpha_n)} \exp\left(-\tfrac{9}{4}\alpha_n^2 \xi\right) \quad (37)$$

The eigenvalues α_n are positive roots of $J_{-\frac{1}{3}}(\alpha) = 0$, tabulated, e.g. in [8]. Figure 6 shows some of the dimensionless temperature profiles.

The Nusselt number at the stationary plate is:

$$Nu_{sp} = \frac{9}{8} \frac{\sum_{n=1}^{\infty} G_n \exp\left(-\tfrac{9}{4}\alpha_n^2 \xi\right)}{\sum_{n=1}^{\infty} (G_n/\alpha_n^2) \exp\left(-\tfrac{9}{4}\alpha_n^2 \xi\right)} \quad (38)$$

where

$$G_n = 1/[\alpha_n^{\frac{4}{3}} J_{\frac{1}{3}}^2(\alpha_n)]. \quad (39)$$

The graphical form of equation (38) is shown in Fig. 7. Since there is no heat transfer at the moving plate, $Nu_{mp} = 0$.

Case D

The temperature distribution is found to be:

$$\vartheta(\eta, \xi) = 2\sqrt{(\eta)} \sum_{n=1}^{\infty} \frac{J_{-\frac{1}{3}}(\alpha_n \eta^{\frac{3}{2}})}{\alpha_n J_{\frac{2}{3}}(\alpha_n)} \times \exp\left(-\tfrac{9}{4}\alpha_n^2 \xi\right). \quad (40)$$

The eigenvalues α_n are positive roots of $J_{-\frac{1}{3}}(\alpha) = 0$, [8]. The temperature profiles are shown in Fig. 8.

The Nusselt number at the upper moving plate is:

$$Nu_{mp} = \frac{9}{8} \frac{\sum_{n=1}^{\infty} \exp\left(-\tfrac{9}{4}\alpha_n^2 \xi\right)}{\sum_{n=1}^{\infty} \alpha_n^{-2} \exp\left(-\tfrac{9}{4}\alpha_n^2 \xi\right)}. \quad (41)$$

This relation is plotted in Fig. 9.

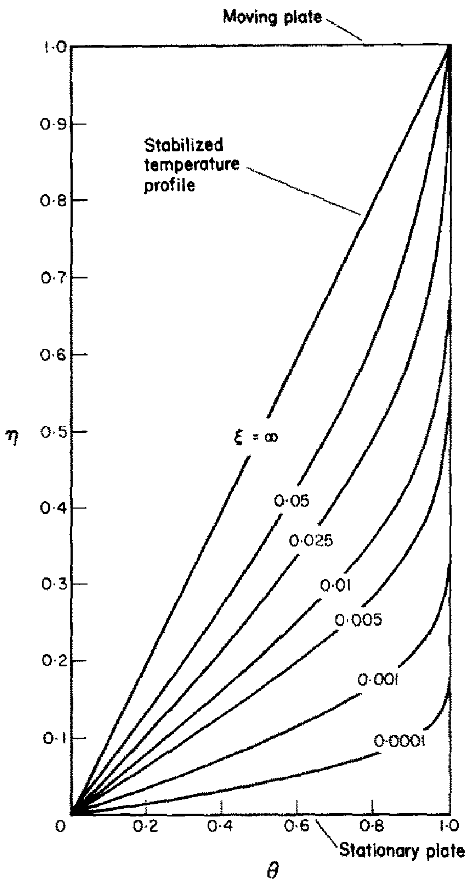

FIG. 4. Dimensionless temperature profiles, boundary condition Case B.

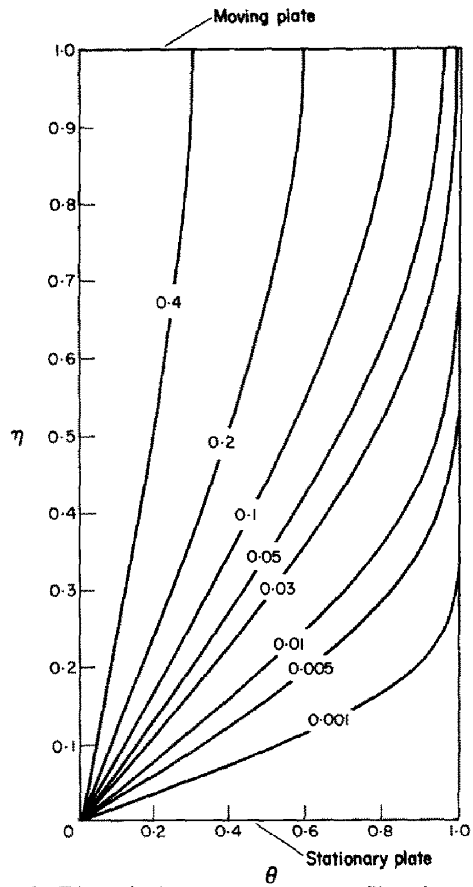

FIG. 6. Dimensionless temperature profiles, boundary condition Case C.

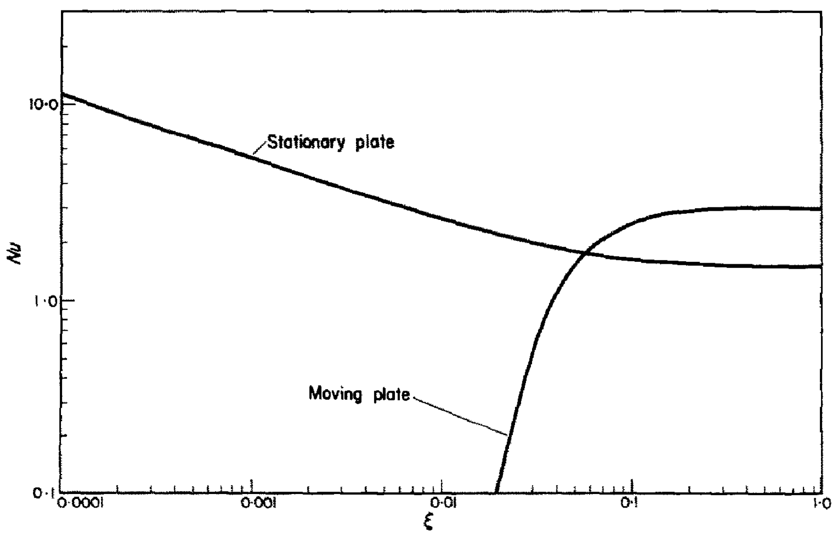

FIG. 5. Variation of the local Nusselt number with longitudinal distance, boundary condition Case B.

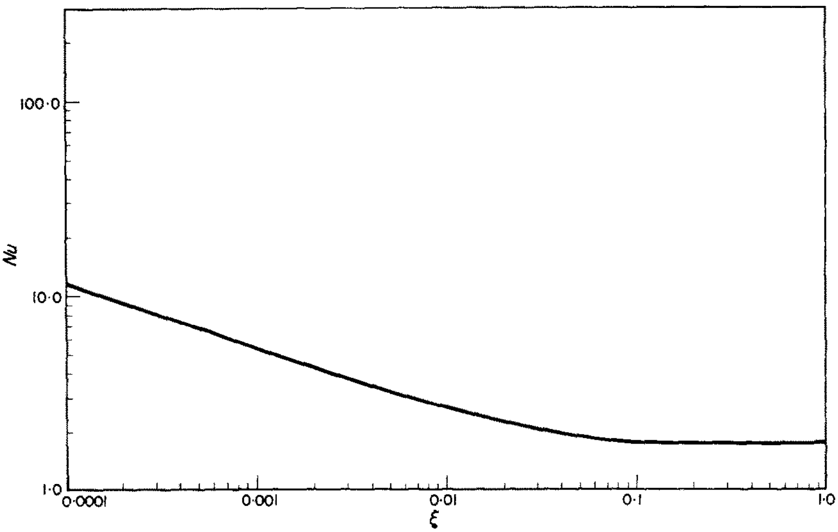

FIG. 7. Local Nusselt number at the stationary plate vs. ξ, boundary condition Case C.

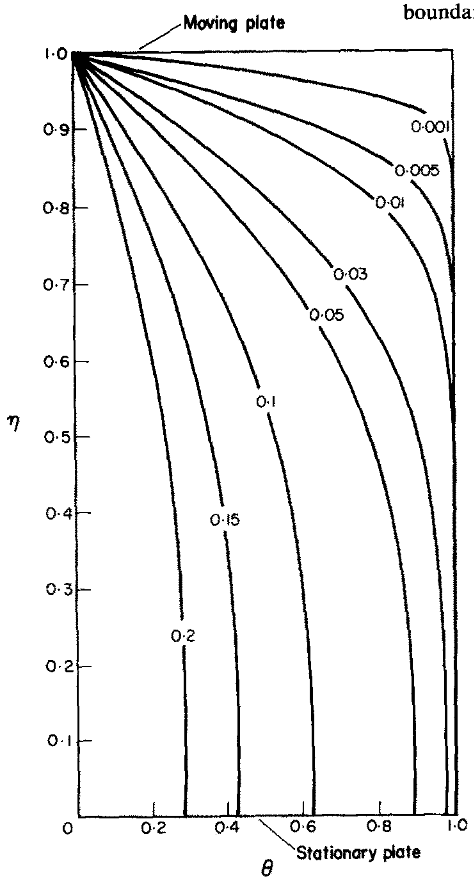

FIG. 8. Dimensionless temperature profiles, boundary condition Case D.

LIMITING VALUES OF THE NUSSELT NUMBER

One of the characteristic features of laminar forced convection heat transfer in ducts of constant cross section is, that, as fully developed velocity and temperature profiles are approached, the Nusselt number converges to a constant value. These limiting values are found from the expressions defining the Nusselt numbers for very large values of the ξ variable; e.g. for boundary condition, Case C the limiting value of the Nusselt group at the lower stationary plate is obtained from equation (38) as:

$$Nu_{sp}(\xi \to \infty) = \tfrac{9}{8}\alpha_1^2. \qquad (42)$$

Recalling that the first positive root of $J_{-\frac{2}{3}}(\alpha) = 0$ is $\alpha_1 = 1{\cdot}243$ there is

$$Nu_{sp}(\infty) = 1{\cdot}738. \qquad (43)$$

Limiting values for other boundary condition cases are listed in Table 1.

Table 1. Limiting values of the Nusselt number

Boundary condition Case		A	B	C	D
Limiting Nusselt number	Nu_{mp}	5·852	3·000	0	3·946
	Nu_{sp}	3·626	1·500	1·738	0

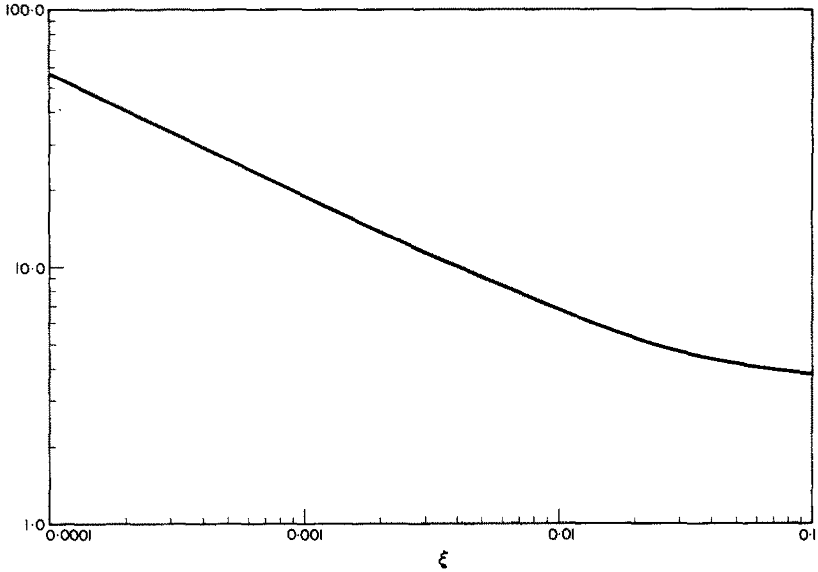

FIG. 9. Local Nusselt number at the moving plate vs. ξ, boundary condition Case D.

CONCLUSION

Exact analytical solutions of the energy equation in terms of temperature profiles and Nusselt numbers have been obtained for the pure Couette flow and heat transfer between parallel plates. An inspection of the temperature profile developments shown graphically reveals the fact that increasing convection at the vicinity of the moving plate results in intensified energy transport thereon. Similarly the heat-transfer rate, expressed in terms of the dimensionless Nusselt group, exhibit higher values at the moving plate. These conclusions are true for all the cases considered. Solutions obtained in this paper may serve as a reliable reference to check techniques developed for the treatment of more complicated problems associated with Couette flow of Newtonian as well as non-Newtonian fluids between parallel plates or narrow-gap coaxial cylinders.

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Résumé—On donne une solution analytique exact de l'équation de l'énergie pour l'écoulement de Couette laminaire en régime permanent d'un fluide visqueux incompressible entre des plaques parallèles. Les propriétés thermiques du fluide sont supposées indépendantes de la température. On considère quatre cas de conditions aux limites. Celles-ci consistent en des échelons unitaires de température sur l'une des plaques ou sur les deux ou un échelon unitaire de température sur une plaque, l'autre plaque étant complètement isolée. Le développement spécial des profils de température est exposé graphiquement. La variation du nombre de Nusselt local avec la distance longitudinale est présentée ainsi que les valeurs limites pour la région de la stabilisation thermique complète.

Zusammenfassung—Für die laminare, stationäre Couette-Strömung einer inkompressiblen, viskosen Flüssigkeit zwischen parallelen Platten wird die exakte analytische Lösung der Energiegleichung angegeben.

Die thermophysikalischen Eigenschaften der Flüssigkeit werden als temperaturunabhängig angenommen. Es werden 4 Fälle von Randbedingungen betrachtet. Diese umfassen schrittweise Temperaturerer-höhungen an einer oder an geiden Platten, bzw. schrittweise Temperaturerhöhung an einer Platte, während die andere vollständig isoliert ist. Die Temperaturprofile werden auf graphischem Wege ermittelt. Die Änderung der örtlichen Nusselt-Zahl in Längsrichtung und die Grenzwerte für das Gebiet der thermisch voll ausgebildeten Strömung werden angegeben.

Аннотация—Приводится точное аналитическое решение уравнения энергии для установившегося ламинарного течения Куэтта несжимаемой вязкой жидкости между параллельными пластинами в допущении независимости теплофизических свойств жидкости от температуры. Рассматриваются четыре типа граничных условий: одиночный температурный импульс на одной или обеих пластинах или импульс температуры на одной пластине, в то время как вторая пластина полностью теплоизолирована. Температурные профили представлены графически. Показано изменение локальных величин критерия Нуссельта в зависимоити от продольного расстояния и предельных значений для области полной термической стабилизации.